Quick Tools for Stochastic Tolerance Analysis

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Abstract — Tolerance analysis of electromagnetic devices during design phase is typically performed using sensitivity analysis or approximate statistical characterization to predict (or assess) the performance of devices before production. Both methods present well known advantages and drawbacks. A quick tool for statistical analysis, combining both methods, is presented in this digest, based on approximate expressions for statistical moments of performance functions, taking advantage of their Taylor expansion. The approach is demonstrated in this digest by assessing the uniformity of the magnetic field created by a pair of Helmotz coils, affected by manufacturing and assembly errors.

I. INTRODUCTION

In the early design phases of electromagnetic devices, limited complexity models must be used, mainly focused on the description of their "nominal" behaviour. Unfortunately, in the practical realisations of devices, due to construction and assembling tolerances, the actual behaviour differs from the optimal one; in addition, during the normal operations, aging and unpredicted effects induce differences in the device operations, which may alter significantly its use [1-3]. In such cases, performance degradation has to be expected; consequently, tolerance assessment or even robust design must be considered before manufacturing phase. On the other hand, the treatment of the assembling tolerances implies a relevant impact on the computational cost because the local behaviour of the performance function $\mathscr{F}$ must be analysed for each design solution (see for instance [1]). The increase of the computational load can become too high for the available computing resources, especially in the robust design, when the number of "uncertain" parameters may be high. Therefore, techniques for the reduction of the computational burden are crucial.

Aim of the paper is to propose a strategy to reduce the computational cost of robust optimization techniques by a preliminary sensitivity analysis of the device performance with respect to parameters, both included into design parameters set or not, and by the successive application of analytical expression for stochastic moments of the device performance function.

In this digest, first the required tools will be presented, i.e. sensitivity analysis in Sect. II and formulas for statistical analysis in Sect. III. Finally, to show effectiveness of proposed approach, an example of performance assessment of a simple electromagnetic device (a pair of Helmotz coils) is presented in Sect. IV. Helmotz coils have been chosen since this device is very sensitive to some of the design parameters; in addition, the performance function can be expressed in analytical form. The effectiveness of the approach in estimating average, standard deviation and worst case of the performance figure is demonstrated by comparing results to the same figures found using MonteCarlo approach.

In the full paper, a comparison of the proposed tools with alternative estimators (such as those provided by unscented transform) will be proposed. In addition, application of the method to more complex electromagnetic devices (such as high field magnets for Nuclear Magnetic Resonance devices, or for Fusion Reactors), where parameters uncertainty are correlated each to the others, will also be proposed.

II. PERFORMANCE SENSITIVITY

In the optimal design of electromagnetic devices, the degrees of freedom (DOF) available to the designer are customarily represented using an array $\underline{p}$. In this digest, for the sake of exposition, it will be assumed that all DOF are real numbers, i.e. $\underline{p}\in\Re^N$, where $\Re$ is the real field, and N is the number of DOF. The DOF array corresponding to the nominal values eventually found as the result of an optimal design procedure will be noted as $\underline{p}_0$, while uncertainties on DOF unavoidably rising as a result of manufacturing and assembly processes of real world devices will be noted as $\delta\underline{p}$, and $\underline{p}=\underline{p}_0+\delta\underline{p}$.

On the other hand, the performance of the device could be defined also in terms of other parameters, not available for optimal choice in the design phase, yet defining the device performance. An example of such "Uncontrolled Parameters" (UP) could be the magnetic permeability of iron parts in magnetic devices. These parameters will be noted as $\underline{q}$ ($\underline{q}\in\Re^M$, where M is the number of UP), their nominal values as $\underline{q}_0$, and their uncertainty with $\delta\underline{q}$.

As a consequence of what exposed above, the device performance will be expressed through a function $\mathscr{F}$:

$$\mathscr{F}(\underline{p}, \underline{q}) : \Re^N\times\Re^M\rightarrow\Re \tag{1}$$

The uncertainties $\delta\underline{p}$ and $\delta\underline{q}$ are in general random variables, each with its own statistical distribution, defined accordingly to the tolerance ranges. On the other hand, it could be reasonably expected that their relative values are rather small, and their effect on the device performance could be expressed by using the Taylor expansion of $\mathscr{F}$ with respect to $\delta\underline{p}$ and $\delta\underline{q}$:

$$\begin{aligned}\mathscr{F}\left(\underline{p}_0+\delta\underline{p},\underline{q}_0+\delta\underline{q}\right)=&\mathscr{F}\left(\underline{p}_0,\underline{q}_0\right)+\\ &\underline{S}_p\left(\underline{p}_0,\underline{q}_0\right)\delta\underline{p}+\underline{S}_q\left(\underline{p}_0,\underline{q}_0\right)\delta\underline{q}+o\left(\left|\delta\underline{p}\right|^2,\left|\delta\underline{q}\right|^2\right)\end{aligned} \tag{2}$$

The arrays $\underline{S}_p$ and $\underline{S}_q$ are called "sensitivity arrays" (with respect to DOF and UP respectively); they can be computed using interpolation, or partial first order derivatives with respect to δp_i or δq_i, or, finally, using covariance computation. Note that different interpretation could drive to different values, but the relevant information is in the relative ranking of sensitivities rather than on their precise values. In the following, the elements of the sensitivity array are computed using spline interpolation.

III. QUICK TOOLS FOR STATISTICAL ANALYSIS OF TOLERANCES

If expansion (2) can be accepted within reasonable accuracy limits, the uncertainty of the performance function in a neighborhood of the nominal design can be approximated by a linear function $\underline{S}_p\delta\underline{p}+S_q\delta\underline{q}$, whose statistical moments can be expressed in closed form once statistical moments of $\delta\underline{p}$ and $\delta\underline{q}$ are known [4]. If this is not the case, higher order terms must be included, without lack of efficiency of the approach. As an example, average $<\mathcal{F}>$ and standard deviation $\sigma_{\mathcal{F}}$ are reported, for the case of uncorrelated tolerances:

$$\langle\mathcal{F}\rangle=\mathcal{F}\left(\underline{p}_0,\underline{q}_0\right)+\sum_i\left.\frac{\partial^2\mathcal{F}}{\partial p_i^2}\right|_{\underline{p}_0,\underline{q}_0}\sigma_{q_i}^2+\sum_i\left.\frac{\partial^2\mathcal{F}}{\partial q_i^2}\right|_{\underline{p}_0,\underline{q}_0}\sigma_{q_i}^2 \qquad (3)$$

$$\sigma_{\mathcal{F}}=\sqrt{\sum_i S_{p_i}^2\sigma_{p_i}^2}+\sqrt{\sum_i S_{q_i}^2\sigma_{q_i}^2}$$

Note that using (3) makes the evaluation of statistical properties of $\mathcal{F}$ a quick task, compatible also with computational efforts required in robust optimization.

Eq. (3) includes also a 2nd order correction term for average, required in the case of perturbations near optimal values; a more complete set of approximated formulas will be used in the full paper [4].

A further figure for which a simple estimation can be easily recovered from sensitivities array is the worst case. In the hypothesis that (2) is valid in the full tolerances range, the device configuration providing the worst performance function is achieved, in the case of unconstrained DOF and UP, at one of the extreme points of the hyper-interval defined by tolerance ranges around the nominal configuration. The corresponding performance function in this case can be estimated as:

$$\mathcal{F}_{WC}=\sum_i\left|S_{p_i}\right|\max(\delta p_i)+\sum_i\left|S_{q_i}\right|\max(\delta q_i) \qquad (4)$$

In the case of constrained tolerances, a more sophisticated algorithm, called TAQS (Tolerance Analysis Quick Solver) must be applied. This case will be described in the full paper.

IV. EXAMPLE OF APPLICATION

In order to show capabilities of the proposed approach, a pair of Helmotz coils is analyzed as a test device. In Helmotz coils, radius of coils must be equal, and each equal to the distance between coils ($R_{c1}=R_{c2}=2^*Z_{c1}=2^*Z_{c2}$, see Fig. 1). This is no more valid when the effect if tolerances is considered, and parameters must be considered each different form the others.

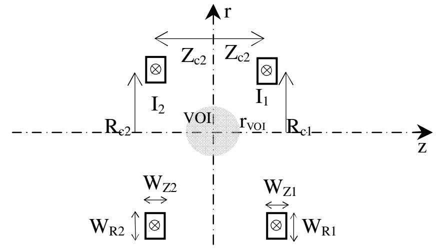

Fig. 1 – A pair of Helmotz coils. In the figure, coordinates of the current baricentre (R_c and Z_c) are indicated, together with radial and axial thickness of the coils (W_R, W_Z)

In this case study, the coils will be assumed massive, with a square footprint. The performance of the devices is expressed through the uniformity of the magnetic field, expressed as:

$$\mathcal{F}\left(\underline{p},\underline{q}\right)=\frac{B_z\left(0,0;\underline{p},\underline{q}\right)-B_z\left(0,r_{VOI};\underline{p},\underline{q}\right)}{B_z\left(0,0;\underline{p},\underline{q}\right)}\times10^6\ p.p.m. \qquad (5)$$

where $B_z(r,z)$ is the axial component of the flux density, and r_{VOI} is the radius of the spherical volume where uniformity is required. The coils have been optimized with respect to radius in order to achieve a desired level of uniformity (7.0 p.p.m.), while cross section dimensions W_R and W_Z have not been included among design parameters. As a consequence, $\delta\underline{p}=(\delta R_c, \delta Z_c)$, and $\delta\underline{q}=(\delta W_R, \delta W_Z)$.

The nominal values are R_C=0.2 m, and $W_R=W_Z$=0.01 m. Tolerances on all parameters have been assumed normally distributed around nominal value, with a standard deviation equal to 1.2% of the nominal value (corresponding to 90% of cases falling inside the tolerance range), except for the radius, for which a tolerance of 50 µm has been assumed, since uniformity is very sensitive to this parameter. Results of the tolerance analysis are reported in Table I, for both proposed approach and MonteCarlo standard analysis, using 10000 uniformity evaluations. Note that, since performance can only deteriorate due to tolerances, second order average estimates must be used.

TABLE I – RESULTS OF STATISTICAL ANALYSIS

Figure	Quick Approach	MonteCarlo
Average Uniformity	7.31 p.p.m.	7.68 p.p.m.
Standard Deviation	5.23 p.p.m.	4.64 p.p.m.
Worst Case	24.13 p.p.m.	18.85 p.p.m.

V. REFERENCES

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